

1. BACKGROUND

The dimensioning process of electronic circuits often results in non-standard values for passive components like resistors, capacitors, or inductors. This can be a critical issue particularly in connection with electronic filters, in which case even small relative deviations of component values from their theoretical values can significantly affect the circuit behaviour.

This raises the fundamental question how, for example, an arbitrary resistance value can be realised or at least closely approximated with standard resistors. Two commonly applied strategies are based on multiple standard resistors which are either connected in series or in parallel; and in case of just two resistors, this is already everything that is possible. When allowing more than two resistors, however, there are also a few more generalised topologies which can be of practical interest.

Another, closely related question arises when a given ratio between the values of two components is to be realised. This is of interest, for example, in connection with voltage dividers, in which case the ratio between two resistance values is the crucial factor rather than the resistance values themselves. Such an arbitrary ratio can be realised or approximated by simply picking an appropriate pair of standard values or, if this is not accurate enough, by using one or two combinations of standard values.

Unfortunately, the necessary calculations to find optimum solutions are not as simple or straightforward as it might seem at first glance. This is because the underlying problems belong to the class of so-called integer optimisation problems, which are inherently hard to solve and, even worse, for which no efficient general-purpose algorithms are known.

Hence, a relatively straightforward way to deal with this is to provide concise, pre-calculated tables of feasible combinations and ratios, and this is exactly the purpose of this collection.

2. TABLES OF COMBINATIONS

This collection of tables deals with combinations of values from the so-called E series of preferred values. These values are commonly used in connection with passive components such as resistors, capacitors, or inductors. The different E series are specified in IEC Standard 60063.

The following two basic types of combinations are addressed:

- Combinations of type A: $X = x_1 + x_2 + \dots + x_n$
- Combinations of type B: $1/X = 1/x_1 + 1/x_2 + \dots + 1/x_n$

Here, x_1, x_2, \dots, x_n are n arbitrary values taken from a particular E series, and X is the resulting value of the respective combination.

Thus, combinations of type A apply to series connections of resistors, to series connections of inductors, and to parallel connections of capacitors. Similarly, combinations of type B apply to parallel connections of resistors, to parallel connections of inductors, and to series connections of capacitors; the latter two cases, however, are of minor practical importance. Note that combinations of less than n values are seamlessly included, in which case some of the x values are either zero (type A) or infinite (type B).

In addition, for the special case of exactly three values, the following two mixed combinations are also addressed:

- Combinations of type C: $X = x_1 + 1 / (1/x_2 + 1/x_3)$
- Combinations of type D: $1/X = 1/x_1 + 1 / (x_2 + x_3)$

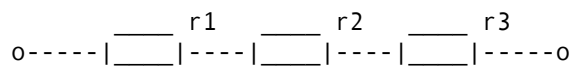
These two combinations are primarily of interest for resistors, in which case combinations of type C apply to extended series connections and combinations of type D to extended parallel connections of one plus two resistors.

In order to limit the potentially infinite number of possible combinations to a reasonable amount, combinations where the ratio between the largest finite value and the smallest non-zero value would exceed an upper limit of 100 are excluded. Such an upper limit corresponds to nominal component tolerances in the order of 1% for the dominant values of a combination, which should be sufficient for most cases of interest.

For convenience and ease of use, the resulting tables of feasible combinations are sorted in ascending order of the resulting values X, and these values are normalised such that they generally fall into the base decade from 100 to 1000, thus $100 \leq X < 1000$. Of course, all table entries can be arbitrarily scaled up or down by factors of 10, 100, 1000, and so forth.

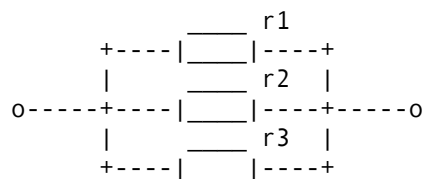
This is best illustrated with combinations of three resistors that have resistance values of r1, r2, and r3, a total resistance value of R, and a maximum allowed ratio of M.

- Combinations of type A:



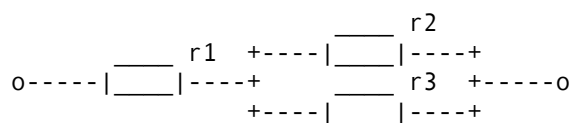
$$R = r1 + r2 + r3$$

- Combinations of type B:



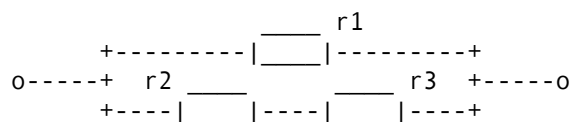
$$1/R = 1/r1 + 1/r2 + 1/r3$$

- Combinations of type C:



$$R = r1 + 1 / (1/r2 + 1/r3)$$

- Combinations of type D:



$$1/R = 1/r1 + 1 / (r2 + r3)$$

Of course, all four combinations are subject to the constraints $100 \leq R < 1000$ and $1 \leq \max(r1,r2,r3) / \min(r1,r2,r3) \leq M$. Note also that the number of values actually used in a basic combination of type A or B can vary from n = 1 (just a single value) up to n = 5, depending on the considered E series.

The following E series are explicitly addressed:

- E6 series: 6 values per decade, combinations up to n = 5
- E12 series: 12 values per decade, combinations up to n = 4
- E24 series: 24 values per decade, combinations up to n = 3
- E48 series: 48 values per decade, combinations up to n = 2
- E48+E24 series: 48 + 21 = 69 values per decade, combinations up to n = 2
- E96 series: 96 values per decade, combinations up to n = 2
- E96+E24 series: 96 + 18 = 114 values per decade, combinations up to n = 2

As regards the combined E48+E24 and E96+E24 series, it is important to note that most E24 values are, for historical reasons, neither contained in the E48 nor in the E96 series. Therefore, these combined pseudo-series are explicitly addressed in separate tables to make this collection as versatile as possible.

All tables also contain a supplementary column holding the normalised relative uncertainty (NRU) value of the respective combination. It is calculated via the common error propagation formula for independent variables and gives an estimate for the mean relative uncertainty of the resulting value X of a combination in relation to the mean relative uncertainties of its x values. This estimate is valid under the simplifying assumption that the x values are statistically independent and normally distributed about their nominal values and that they have identical relative uncertainties, in which case $1/\sqrt{n} \leq \text{NRU} \leq 1$. Incidentally, these theoretical NRU values represent upper bounds for all symmetrical distributions that are strictly bounded by the given tolerance bands, which should normally be the case in the current context.

In effect, this means that the relative dispersion of the resulting value X of a combination is somewhat reduced compared to the relative dispersions of its individual x values. The worst-case behaviour in the sense of a strict tolerance value for X is not improved, though. Note, however, that the above independence assumption may already be violated when a combination is made up of components from the same production batch, just to name one potential caveat.

3. TABLES OF RATIOS

This additional set of tables deals with ratios between two values or combinations of values from the E series of preferred values. In order not to render these tables too complex, however, combinations are restricted to the two basic types A and B described above, and ratios involving both types of combinations are not taken into consideration.

The ratio between two such values or combinations of values, respectively, can then be expressed by the following fraction:

$$- Z = X / Y$$

The numerator X and the denominator Y of this fraction are in turn combinations of type A or B. Thus, for combinations of type A:

$$- X = x_1 + x_2 + \dots + x_{nx}$$

$$- Y = y_1 + y_2 + \dots + y_{ny}$$

And for combinations of type B:

$$- 1/X = 1/x_1 + 1/x_2 + \dots + 1/x_{nx}$$

$$- 1/Y = 1/y_1 + 1/y_2 + \dots + 1/y_{ny}$$

Here, x_1, x_2, \dots, x_{nx} are n_x arbitrary values from a particular E series, and X is the resulting value of the respective combination. Similarly, y_1, y_2, \dots, y_{ny} are n_y arbitrary values from the same E series with the resulting value Y. Z is the ratio between the two resulting values.

Combinations of type A apply again to series connections of resistors, to series

connections of inductors, and to parallel connections of capacitors. Similarly, combinations of type B apply to parallel connections of resistors, to parallel connections of inductors, and to series connections of capacitors; the latter two cases are of minor practical importance, though. Combinations of less than n_x or n_y values are seamlessly included, in which case some of the x or y values are either zero (type A) or infinite (type B).

In order to limit the potentially infinite number of possible numerator or denominator combinations to a reasonable amount, combinations where the ratio between the largest finite value and the smallest non-zero value would exceed an upper limit of 100 are again excluded.

For convenience and ease of use, the resulting tables of feasible ratios are sorted in ascending order of the resulting ratios Z , and these ratios are normalised such that they generally fall into the base range from 1 to 10, thus $1 \leq Z < 10$. Moreover, the numerators X are further normalised such that they fall into the base decade from 100 to 1000, thus $100 \leq X < 1000$. This does not introduce any loss of generality because X and Y can be independently scaled up or down by factors of 10, 100, 1000, and so forth, which allows to adjust the magnitudes of the numerators and denominators and, in turn, those of the resulting ratios as well.

All this is best illustrated with ratios that are realised using three resistors in total, one single resistor and a two-resistor combination. The underlying resistance values are r_{x1} and r_{x2} for the numerator and r_{y1} and r_{y2} for the denominator, respectively, with resulting resistance values of R_X and R_Y . The maximum allowed ratio between the largest and smallest value of a combination is M .

- Ratios involving combinations of type A:

$$o \text{-----} \left| \frac{\quad}{\quad} r_{x1} \right| \text{-----} o \quad \text{and} \quad o \text{-----} \left| \frac{\quad}{\quad} r_{y1} \right| \text{-----} \left| \frac{\quad}{\quad} r_{y2} \right| \text{-----} o$$

$$R_X = r_{x1}$$

$$R_Y = r_{y1} + r_{y2}$$

$$o \text{-----} \left| \frac{\quad}{\quad} r_{x1} \right| \text{-----} \left| \frac{\quad}{\quad} r_{x2} \right| \text{-----} o \quad \text{and} \quad o \text{-----} \left| \frac{\quad}{\quad} r_{y1} \right| \text{-----} o$$

$$R_X = r_{x1} + r_{x2}$$

$$R_Y = r_{y1}$$

- Ratios involving combinations of type B:

$$o \text{-----} \left| \frac{\quad}{\quad} r_{x1} \right| \text{-----} o \quad \text{and} \quad o \text{-----} + \left| \frac{\quad}{\quad} r_{y1} \right| \text{-----} +$$

$$+ \left| \frac{\quad}{\quad} r_{y2} \right| \text{-----} +$$

$$R_X = r_{x1}$$

$$1/R_Y = 1/r_{y1} + 1/r_{y2}$$

$$o \text{-----} + \left| \frac{\quad}{\quad} r_{x1} \right| \text{-----} +$$

$$+ \left| \frac{\quad}{\quad} r_{x2} \right| \text{-----} + \quad \text{and} \quad o \text{-----} \left| \frac{\quad}{\quad} r_{y1} \right| \text{-----} o$$

$$1/R_X = 1/r_{x1} + 1/r_{x2}$$

$$R_Y = r_{y1}$$

The resulting ratios $Z = R_X / R_Y$ and their numerators R_X are subject to the constraints $1 \leq Z < 10$ and $100 \leq R_X < 1000$. Moreover, the combinations are

subject to the additional constraints $1 \leq \max(rx1,rx2) / \min(rx1,rx2) \leq M$ and $1 \leq \max(ry1,ry2) / \min(ry1,ry2) \leq M$, respectively. Note, however, that the total number of values that is actually used for realising an arbitrary ratio can vary from $n = 2$ (just one pair of values) up to $n = nx + ny = 4$, depending on the considered E series; nx and ny are the number of numerator and denominator values, respectively.

The following E series are explicitly addressed:

- E6 series: ratios up to $n = nx + ny = 4$
- E12 series: ratios up to $n = nx + ny = 3$
- E24 series: ratios up to $n = nx + ny = 3$
- E48 series: ratios of two values
- E48+E24 series: ratios of two values
- E96 series: ratios of two values
- E96+E24 series: ratios of two values

The combined E48+E24 and E96+E24 series are treated as described above.

4. PACKAGES

The tables contained in this collection are made available in five separate packages, each of which containing the set of files for a particular E series. For simplicity, however, the E48 and the E48+E24 files share the same package, just like the E96 and the E96+E24 files. Each such package is stored in a ZIP archive to save space as well as download time. Besides, this also guarantees the integrity of the contained text files when transferred across different platforms.

The whole collection consists of the following seven files:

- Read_Me.pdf
- Read_Me.txt

- E6_Combinations.zip
- E12_Combinations.zip
- E24_Combinations.zip
- E48_Combinations.zip
- E96_Combinations.zip

The read-me file is included in PDF and in text form, for convenience. Unpacking the ZIP files on current operating systems such as macOS, Unix, or Windows ought to be straightforward. Instructions on how to do this can be found, for example, at <http://www.wikihow.com/wikiHowTo?search=unzip>.

After unpacking, each of the above ZIP files expands to a single folder (or directory). The corresponding five folders should then contain the following files:

Folder 'E6_Combinations' (2.8 MB):

- E6_Read_Me.pdf
- E6_Read_Me.txt

- E6_Combinations_2_A.txt
- E6_Combinations_2_B.txt
- E6_Combinations_3_A.txt
- E6_Combinations_3_B.txt
- E6_Combinations_3_C.txt
- E6_Combinations_3_D.txt
- E6_Combinations_4_A.txt
- E6_Combinations_4_B.txt

- E6_Combinations_5_A.txt
- E6_Combinations_5_B.txt

- E6_Ratios_2.txt
- E6_Ratios_3_A.txt
- E6_Ratios_3_B.txt
- E6_Ratios_4_A.txt
- E6_Ratios_4_B.txt

- GNU_GPL_V3.txt
- GNU_LGPL_V3.txt

Folder 'E12_Combinations' (4.8 MB):

- E12_Read_Me.pdf
- E12_Read_Me.txt

- E12_Combinations_2_A.txt
- E12_Combinations_2_B.txt
- E12_Combinations_3_A.txt
- E12_Combinations_3_B.txt
- E12_Combinations_3_C.txt
- E12_Combinations_3_D.txt
- E12_Combinations_4_A.txt
- E12_Combinations_4_B.txt

- E12_Ratios_2.txt
- E12_Ratios_3_A.txt
- E12_Ratios_3_B.txt

- GNU_GPL_V3.txt
- GNU_LGPL_V3.txt

Folder 'E24_Combinations' (12.8 MB):

- E24_Read_Me.pdf
- E24_Read_Me.txt

- E24_Combinations_2_A.txt
- E24_Combinations_2_B.txt
- E24_Combinations_3_A.txt
- E24_Combinations_3_B.txt
- E24_Combinations_3_C.txt
- E24_Combinations_3_D.txt

- E24_Ratios_2.txt
- E24_Ratios_3_A.txt
- E24_Ratios_3_B.txt

- GNU_GPL_V3.txt
- GNU_LGPL_V3.txt

Folder 'E48_Combinations' (1.3 MB):

- E48_Read_Me.pdf
- E48_Read_Me.txt

- E48_Combinations_2_A.txt
- E48_Combinations_2_B.txt
- E48_E24_Combinations_2_A.txt
- E48_E24_Combinations_2_B.txt

- E48_Ratios_2.txt

- E48_E24_Ratios_2.txt
- GNU_GPL_V3.txt
- GNU_LGPL_V3.txt

Folder 'E96_Combinations' (3.7 MB):

- E96_Read_Me.pdf
- E96_Read_Me.txt
- E96_Combinations_2_A.txt
- E96_Combinations_2_B.txt
- E96_E24_Combinations_2_A.txt
- E96_E24_Combinations_2_B.txt
- E96_Ratios_2.txt
- E96_E24_Ratios_2.txt
- GNU_GPL_V3.txt
- GNU_LGPL_V3.txt

All read-me files are again included in PDF and in text form. The individual tables of feasible combinations or ratios are stored in text files that have systematically coded file names of the form:

- <series>_Combinations_<n>_<type>.txt
- <series>_Ratios_<n>.txt
- <series>_Ratios_<n>_<type>.txt

The string <series> specifies the underlying E series or the combined pseudo-series, <n> specifies the number or the maximum number, respectively, of values used in a combination or for a ratio, and <type>, if applicable, specifies the type of combinations.

Each of these files also contains information about its original size in bytes, and it is recommended to cross-check this if a problem with one of the files is encountered, before contacting the author.

The last two files in each folder are the GNU General Public License (GPL) and the GNU Lesser General Public License (LGPL). These files contain the detailed license agreements under which the accompanying tables are being released.

All text files included in these packages are Unix plain text files which use single line-feed (LF) characters as line terminators; the various tables also assume a fixed tab spacing of eight characters. Thus, a monospaced font and proper tab settings are recommended for best readability. On Windows, it may be necessary, too, to replace LF with CR LF (which will increase the file sizes), or to use a compatible text viewer or editor.

5. FEEDBACK

This collection of tables is developed and maintained by Gert Willmann. Please send comments, questions, or bug reports to my e-mail alias at 'ieee.org'; the mailbox or user-name, respectively, is 'gert.willmann' (this indirect specification merely serves to prevent spam).

Alternatively, or if e-mail doesn't work, my postal address is:

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